

EIGENSYSTEMS OF NON-SELF-ADJOINT OPERATORS

BORIS S. MITYAGIN

The lectures focus on the geometry of systems of eigenvectors of non-self-adjoint operators. The main examples come from analysis of two families of ODE operators:

(a) Hill-Schrödinger operators

$$(1) \quad Ly = -y'' + v(x)y, \quad 0 \leq x \leq \pi, \quad v(x + \pi) = v(x);$$

(b) the harmonic oscillator and its perturbations

$$(2) \quad My = -y'' + x^2y + w(x)y, \quad -\infty < x < \infty.$$

The mini course is based on, but not limited to, the work of the lecturer and his coauthors P. Djakov, J. Adduci, P. Siegl, and J. Viola.

Tentative Schedule.

1. Instability zones (spectral gaps) of an operator (1).

Special attention is given to their asymptotics in the case of a two-term potential

$$v(x) = a \cos(2x) + b \cos(4x).$$

2. Spectral decompositions of Hill operators with trigonometric polynomial potentials.

Complete analysis (combinatorics) is done in the case of potentials

- (i) $ae^{-2ix} + be^{2ix}$;
- (ii) $ae^{-2ix} + Be^{4ix}$;
- (iii) $ae^{-2ix} + Ae^{-4ix} + be^{2ix} + Be^{4ix}$.

3. Root Systems of Perturbations of Harmonic-Oscillator-Type Operator

4. Differential Operators admitting various rates of spectral projection growth

LITERATURE (INCOMPLETE LIST)

- [1] Plamen Djakov and Boris Mityagin. Instability zones of one-dimensional periodic Schrödinger and Dirac operators. *Uspekhi Mat. Nauk*, 61(4(370)):77–182, 2006. In Russian, English transl. in : *Russian Math. Surveys* 61 (4) (2006) 663 – 766.
- [2] Plamen Djakov and Boris Mityagin. Asymptotics of instability zones of the Hill operator with a two term potential. *J. Funct. Anal.*, 242(1):157 – 194, 2007.
- [3] Plamen Djakov and Boris Mityagin. Convergence of spectral decompositions of Hill operators with trigonometric polynomial potentials. *Mathematische Annalen*, 351(3):509–540, 2011.
- [4] Plamen Djakov and Boris Mityagin. Criteria for existence of riesz bases consisting of root functions of Hill and 1D Dirac operators. *J. Funct. Anal.*, 263(8):2300 – 2332, 2012.
- [5] James Adduci and Boris Mityagin. Eigensystem of an L^2 -perturbed harmonic oscillator is an unconditional basis. *Cent. Eur. J. Math.*, 10(2):569–589, 2012.
- [6] Boris Mityagin and Petr Siegl. Root system of singular perturbations of the harmonic oscillator type operators. *Lett. Math. Phys.*, 106(2):147–167, 2016.
- [7] Boris Mityagin. The spectrum of a harmonic oscillator operator perturbed by point interactions. *International Journal of Theoretical Physics*, 54(11):4068–4085, 2015.
- [8] Boris Mityagin, Petr Siegl, and Joe Viola. Differential operators admitting various rates of spectral projection growth. *J. Funct. Anal.*, 272(8):3129 – 3572, 2017.