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The Congruence Subgroup Problem for Automorphism Groups

In its classical setting, the Congruence Subgroup Problem (CSP) asks whether every finite index subgroup of $GL_n(\mathbb{Z})$ contains a principal congruence subgroup of the form $\ker(GL_n(\mathbb{Z}) \to GL_n(\mathbb{Z}/m\mathbb{Z}))$ for some $m \in \mathbb{Z}$. It was known already in the 19th century that for $n = 2$ the answer is negative, and actually $GL_2(\mathbb{Z})$ has many finite index subgroups which do not come from congruence considerations. On the other hand, quite surprisingly, in the sixties it was found out by Mennicke, and separately by Bass-Lazard-Serre, that the answer for $n > 2$ is affirmative. This result was a breakthrough that led to a rich theory which generalized the problem to matrix groups over rings.

Viewing $GL_n(\mathbb{Z}) \cong Aut(\mathbb{Z}^n)$ as the automorphism group of of $\Delta = \mathbb{Z}^n$, one can generalize the CSP to automorphism groups as follows: Let Δ be a group, does every finite index subgroup of $Aut(\Delta)$ contain a principal congruence subgroup of the form: $\ker(Aut(\Delta) \to Aut(\Delta/M))$ for some finite index characteristic subgroup $M \leq \Delta$? Considering this generalization, there are very few results when Δ is non-abelian. For example, only in 2001 Asada proved, using tools from Algebraic Geometry, that $Aut(F_2)$ has an affirmative answer to the CSP, when F_2 is the free group on two generators. For $Aut(F_n)$ when $n > 2$ the problem is still unsettled. On the talk, I will present the problem from a few aspects, and introduce some recent results for non-abelian groups. The main result will assert that while the dichotomy in the abelian case is between $n = 2$ and $n > 2$, when Δ is the free metabelian group on n generators, we have a dichotomy between $n = 2, 3$ and $n > 3$.