

Steklov Mathematical Institute, St. Petersburg, & Chebyshev Laboratory, SPbU

JOINT COLLOQUIUM

Thursday, March 9 at 17:00

Marble Hall, Steklov Mathematical Institute, Fontanka 27, St. Petersburg

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Multidimensional Newton's lemma on integrable domains and monodromy theory

A bounded domain in a Euclidean space defines a (two-valued) function on the space of all affine hyperplanes in it: the volumes cut by the hyperplanes from our domain. A domain is called algebraically integrable if this function is algebraic. The famous Lemma XXVIII from Newton's "Principia" says that there are no integrable domains with smooth boundary in the plane. We show that the same holds for the domains in any even-dimensional space (while for the case of odd dimensions we have the Archimedes' counterexample). The proof is based on the (Picard-Lefschetz) monodromy theory of complex algebraic varieties, and the theory of finite reflection groups.

This integrability problem is a sample of numerous problems of mathematics and physics related with integral representations, in which the methods of Picard-Lefschetz theory give us crucial information on analytical properties (such as existence, ramification, number of, etc) of the functions given by such representations.